

## Modeling Compound Growth in Excel Part 3 : Annuities

Robert Muller  
CS 021 Computers in Management  
Boston College

---

---

---

---

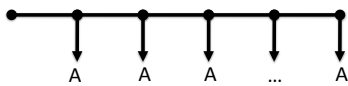
---

---

---

---

### An Annuity is a Sequence of Cash Flows



9/24/09

CS 021 Computers in Management

2

---

---

---

---

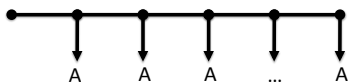
---

---

---

---

### An Annuity is a Sequence of Cash Flows



Examples include mortgages, bonds, rent and loan payments.

9/24/09

CS 021 Computers in Management

3

---

---

---

---

---

---

---

---

### Cash Flows at End of Period (type 0)

A horizontal timeline with a starting point on the left and an ending point on the right. Five downward-pointing arrows are positioned below the timeline at regular intervals. The first arrow is orange and labeled 'pmt'. The second arrow is green and labeled 'pmt'. The third arrow is black and labeled 'pmt'. The fourth arrow is grey and labeled '...'. The fifth arrow is purple and labeled 'pmt'.

$FV(nper, rate, pmt, pv, type)$

9/24/09 CS 021 Computers in Management 4

---

---

---

---

---

---

---

---

### Cash Flows at Start of Period (type 1)

A horizontal timeline with a starting point on the left and an ending point on the right. Five downward-pointing arrows are positioned below the timeline at regular intervals. The first arrow is orange and labeled 'pmt'. The second arrow is green and labeled 'pmt'. The third arrow is black and labeled 'pmt'. The fourth arrow is grey and labeled '...'. The fifth arrow is purple and labeled 'pmt'.

type 0 is the default

9/24/09 CS 021 Computers in Management 5

---

---

---

---

---

---

---

---

### Annuities Have Future and Present Values

A horizontal timeline with a starting point on the left and an ending point on the right. Five downward-pointing arrows are positioned below the timeline at regular intervals. The first arrow is labeled 'A<sub>1</sub>', the second 'A<sub>2</sub>', the third 'A<sub>3</sub>', the fourth '...', and the fifth 'A<sub>n</sub>'.

9/24/09 CS 021 Computers in Management 6

---

---

---

---

---

---

---

---

### Future Value (type 1)

9/24/09 CS 021 Computers in Management 7

---

---

---

---

---

---

---

---

### Future Value (type 1)

$$FV_n = A_1(1+i)^n + A_2(1+i)^{n-1} + \dots + A_n(1+i)^1$$

9/24/09 CS 021 Computers in Management 8

---

---

---

---

---

---

---

---

### Future Value (type 1)

$$FV_n = A(1+i)^n + A(1+i)^{n-1} + \dots + A(1+i)^1$$

9/24/09 CS 021 Computers in Management 9

---

---

---

---

---

---

---

---

## Closed Form

$$FV_n = A(1+i)^n + A(1+i)^{n-1} + \dots + A(1+i)^1$$

We want to eliminate  
the "and so on" ellipsis

9/24/09

CS 021 Computers in Management

10

---

---

---

---

---

---

---

---

## Geometric Series

A sum of the form:

$$\sum_{i=1}^n ar^i = ar^1 + ar^2 + \dots + ar^n$$

9/24/09

CS 021 Computers in Management

11

---

---

---

---

---

---

---

---

## Closed form of a Geometric Series

$$\sum_{i=1}^n ar^i = ar^1 + ar^2 + \dots + ar^n$$

$$(1-r) \sum_{i=1}^n ar^i = (1-r)(ar^1 + ar^2 + \dots + ar^n)$$

9/24/09

CS 021 Computers in Management

12

---

---

---

---

---

---

---

---

## Closed form of a Geometric Series

$$\sum_{i=1}^n ar^i = ar^1 + ar^2 + \dots + ar^n$$

$$(1-r) \sum_{i=1}^n ar^i = (1-r)(ar^1 + ar^2 + \dots + ar^n)$$

9/24/09

CS 021 Computers in Management

13

---

---

---

---

---

---

---

---

## Closed form of a Geometric Series

$$(1-r) \sum_{i=1}^n ar^i = (1-r)(ar^1 + ar^2 + \dots + ar^n)$$

$$(1-r) \sum_{i=1}^n ar^i = ar^1 + ar^2 + \dots + ar^n \\ - ar^2 - \dots - ar^n - ar^{n+1}$$

9/24/09

CS 021 Computers in Management

14

---

---

---

---

---

---

---

---

## Closed form of a Geometric Series

$$(1-r) \sum_{i=1}^n ar^i = ar^1 + ar^2 + \dots + ar^n \\ - ar^2 - \dots - ar^n - ar^{n+1}$$

$$= ar - ar^{n+1}$$

$$= \frac{a(r^{n+1} - r)}{r - 1}$$

9/24/09

CS 021 Computers in Management

15

---

---

---

---

---

---

---

---

### Closed form of a Geometric Series

$$\sum_{i=1}^n ar^i = \frac{a(r^{n+1} - r)}{r - 1}$$

9/24/09

CS 021 Computers in Management

16

---

---

---

---

---

---

---

---

### Future Value

- Plugging  $(1 + i)$  in for  $r$ :

$$\begin{aligned} FV_n &= \frac{A((1 + i)^{n+1} - (1 + i))}{(1 + i) - 1} \\ &= \frac{A((1 + i)^{n+1} - 1 - 1)}{i} \\ &= FV(i, n, -A, 0, 1) \end{aligned}$$

9/24/09

CS 021 Computers in Management

17

---

---

---

---

---

---

---

---

### Example

**Problem** : Andrew's grandparents have put \$500 in a bank account for him every year on his birthday. How much will he have after the payment on his 21<sup>st</sup> birthday if the money grows at 5% per year?

**Answer:**

9/24/09

CS 021 Computers in Management

18

---

---

---

---

---

---

---

---

### Example

**Problem :** Andrew's grandparents have put \$500 in a bank account for him every year on his birthday. How much will he have after the payment on his 21<sup>st</sup> birthday if the money grows at 5% per year?

$$\text{Answer: } = \frac{500((1 + .05)^{21+1} - .05 - 1)}{.05} + 500$$

9/24/09

CS 021 Computers in Management

19

---

---

---

---

---

---

---

---

### Example

**Problem :** Andrew's grandparents have put \$500 in a bank account for him every year on his birthday. How much will he have after the payment on his 21<sup>st</sup> birthday if the money grows at 5% per year?

$$\begin{aligned} \text{Answer: } &= \text{FV}(5\%, 21, -500, 0, 1) + 500 \\ &= \$19,252 \end{aligned}$$

9/24/09

CS 021 Computers in Management

20

---

---

---

---

---

---

---

---

### Allowing for Nonzero PV

$$\begin{aligned} \text{FV} &= \text{PV}(1 + i)^n + \frac{A((1 + i)^{n+1} - i - 1)}{i} \\ &= \text{FV}(i, n, -A, \text{PV}, 1) \end{aligned}$$

9/24/09

CS 021 Computers in Management

21

---

---

---

---

---

---

---

---

### Solving for the Annuity A (pmt)

$$FV = PV(1 + i)^n + \frac{A((1 + i)^{n+1} - i - 1)}{i}$$

$$FV - PV(1 + i)^n = \frac{A((1 + i)^{n+1} - i - 1)}{i}$$

$$A = \frac{i(FV - PV(1 + i)^n)}{(1 + i)^{n+1} - i - 1}$$

9/24/09

CS 021 Computers in Management

22

---

---

---

---

---

---

---

---

### Solving for the Annuity A (pmt)

$$A = \frac{i(FV - PV(1 + i)^n)}{(1 + i)^{n+1} - i - 1}$$

$$= \text{pmt}(\text{rate}^i, \text{nper}^n, \text{PV}, \text{FV}, \text{type})$$

9/24/09

CS 021 Computers in Management

23

---

---

---

---

---

---

---

---

### Example

**Problem :** Andrew would like to withdraw a fixed amount once a month while he is in law school. How much can he withdraw assuming that he wants to save half the principal? Assume the same interest rate but compounding monthly.

**Answer:**

9/24/09

CS 021 Computers in Management

24

---

---

---

---

---

---

---

---



### Example

**Problem :** Andrew would like to withdraw a fixed amount once a month while he is in law school. How much can he withdraw assuming that he wants to save half the principal? Assume the same interest rate but compounding monthly.

**Answer:**

$$\text{Let } X = \frac{500((1 + .05)^{21+1} - .05 - 1)}{.05} + 500$$

---

---

---

---

---

---

---

---

### Example

**Problem :** Andrew would like to withdraw a fixed amount once a month while he is in law school. How much can he withdraw assuming that he wants to save half the principal? Assume the same interest rate but compounding monthly.

**Answer:**

$$\text{Let } X = \frac{500((1 + .05)^{21+1} - .05 - 1)}{.05} + 500$$

$$A = \frac{.05/12(X/2 - X(1 + .05/12)^{36})}{(1 + .05/12)^{36+1} - (.05/12) - 1}$$

---

---

---

---

---

---

---

---

### Example

**Problem :** Andrew would like to withdraw a fixed amount once a month while he is in law school. How much can he withdraw assuming that he wants to save half the principal? Assume the same interest rate but compounding monthly.

**Answer:**

$$\begin{aligned} A1 &= FV(5\%, 21, -500, 0, 1) + 500 \\ &= PMT(5\%/12, 36, -A1, A1/2, 1) \\ &= \$327 \end{aligned}$$

---

---

---

---

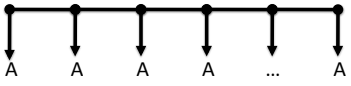
---

---

---

---

### Present Value



A horizontal timeline with six tick marks. Below each tick mark is a downward-pointing arrow labeled 'A'. The fifth tick mark is followed by an ellipsis '...', and the sixth tick mark is also labeled 'A'.

9/24/09 CS 021 Computers in Management 28

---

---

---

---

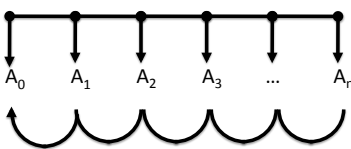
---

---

---

---

### Present Value



A horizontal timeline with six tick marks. Below each tick mark is a downward-pointing arrow labeled  $A_0$ ,  $A_1$ ,  $A_2$ ,  $A_3$ , ...,  $A_n$ . Below the timeline, a series of curved arrows point from each tick mark back to the first tick mark, representing the discounting of each payment to its present value.

9/24/09 CS 021 Computers in Management 29

---

---

---

---

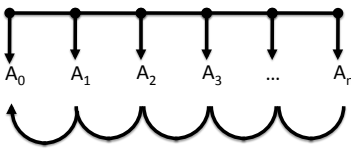
---

---

---

---

### Present Value



A horizontal timeline with six tick marks. Below each tick mark is a downward-pointing arrow labeled  $A_0$ ,  $A_1$ ,  $A_2$ ,  $A_3$ , ...,  $A_n$ . Below the timeline, a series of curved arrows point from each tick mark back to the first tick mark, representing the discounting of each payment to its present value.

$$PV = \frac{A_0}{(1+i)^0} + \frac{A_1}{(1+i)^1} + \dots + \frac{A_n}{(1+i)^n}$$

9/24/09 CS 021 Computers in Management 30

---

---

---

---

---

---

---

---

### Present Value

$$PV = A + \frac{A}{(1+i)} + \dots + \frac{A}{(1+i)^n}$$

$$= \frac{A}{i} \left[ 1 - \frac{1}{(1+i)^n} \right]$$

$$= PV(\text{rate}^i, \text{nper}^n, A, 0, 1)$$

9/24/09

CS 021 Computers in Management

31

---

---

---

---

---

---

---

---

### Example

**Problem** : You want to buy a house for \$500,000. You've saved up 20% and your bank is offering to loan you the remaining 80% for 15 years at 7% annual interest compounded monthly. How much interest will you pay in the 8<sup>th</sup> year?

**Answer**: See the LoanAmortization spreadsheet.

9/24/09

CS 021 Computers in Management

32

---

---

---

---

---

---

---

---