Modeling Compound Growth in Excel Part 3 : Annuities

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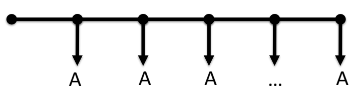
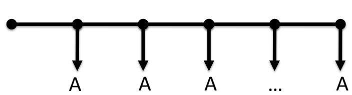
An Annuity is a Sequence of Cash Flows

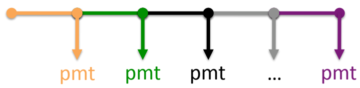


An Annuity is a Sequence of Cash Flows



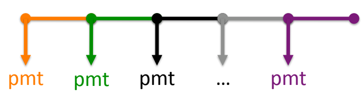
Examples include mortgages, bonds, rent and loan payments.

Cash Flows at End of Period (type 0)



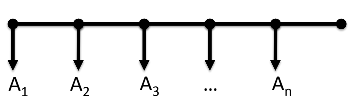
FV(nper, rate, pmt, pv, type)

Cash Flows at Start of Period (type 1)

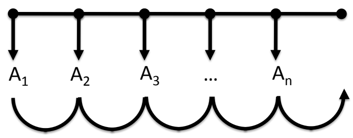


type 0 is the default

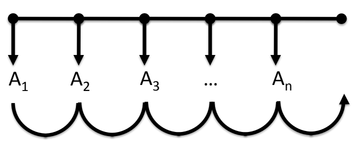
Annuities Have Future and Present Values



Future Value (type 1)

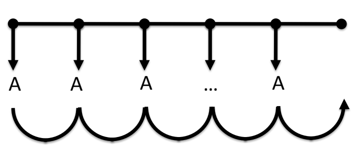


Future Value (type 1)



$$FV_n = A_1(1+i)^n + A_2(1+i)^{n-1} + \ldots + A_n(1+i)^1$$

Future Value (type 1)



$$FV_n = A(1+i)^n + A(1+i)^{n-1} + \ldots + A(1+i)^1$$

Closed Form

$FV_n = A(1+i)^n + A(1+i)^{n-1} + \ldots + A(1+i)^1$

We want to eliminate the "and so on" ellipsis

Geometric Series

A sum of the form:

$$\sum_{i=1}^{n} ar^i = ar^1 + ar^2 + \ldots + ar^n$$

Closed form of a Geometric Series

$$\sum_{i=1}^{n} ar^i = ar^1 + ar^2 + \ldots + ar^n$$

$$(1-r) \sum_{i=1}^{n} ar^i = (1-r)(ar^1 + ar^2 + \ldots + ar^n)$$

Closed form of a Geometric Series

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9/24/09 CS 021 Computers in Management 13

Closed form of a Geometric Series

$$(1-r)\sum_{i=1}^{n} ar^i = (1-r)(ar^1 + ar^2 + \ldots + ar^n)$$

$$(1-r)\sum_{i=1}^{n} ar^i = ar^1 + ar^2 + \ldots + ar^n - ar^2 - \ldots - ar^n - ar^{n+1}$$

9/24/09 CS 021 Computers in Management 14

Closed form of a Geometric Series

$$(1-r)\sum_{i=1}^{n} ar^i = ar^1 + ar^2 + \ldots + ar^n - ar^2 - \ldots - ar^n - ar^{n+1}$$

$$= ar - ar^{n+1}$$

$$= \frac{a(r^{n+1} - r)}{r - 1}$$

9/24/09 CS 021 Computers in Management 15

Closed form of a Geometric Series

$$\sum_{i=1}^{n} ar^i = \frac{a(r^{n+1} - r)}{r - 1}$$

Future Value

- Plugging (1 + i) in for r:

$$FV_n = \frac{A((1+i)^{n+1} - (1+i))}{(1+i) - 1}$$

$$= \frac{A((1+i)^{n+1} - i - 1)}{i}$$

$$= FV(i, n, -A, 0, 1)$$

Example

Problem : Andrew's grandparents have put $500 in a bank account for him every year on his birthday. How much will he have after the payment on his 21st birthday if the money grows at 5% per year?

Answer:

Example

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Answer: $= \dfrac{500((1 + .05)^{21+1} - .05 - 1)}{.05} + 500$

Example

Problem : Andrew's grandparents have put $500 in a bank account for him every year on his birthday. How much will he have after the payment on his 21st birthday if the money grows at 5% per year?

Answer: = FV(5%, 21, -500, 0, 1) + 500
= $19,252

Allowing for Nonzero PV

$$FV = PV(1 + i)^n + \frac{A((1 + i)^{n+1} - i - 1)}{i}$$

= FV(i, n, -A, PV, 1)

Solving for the Annuity A (pmt)

$$FV = PV(1+i)^n + \frac{A((1+i)^{n+1} - i - 1)}{i}$$

$$FV - PV(1+i)^n = \frac{A((1+i)^{n+1} - i - 1)}{i}$$

$$A = \frac{i(FV - PV(1+i)^n)}{(1+i)^{n+1} - i - 1}$$

Solving for the Annuity A (pmt)

$$A = \frac{i(FV - PV(1+i)^n)}{(1+i)^{n+1} - i - 1}$$

= pmt(rate^i, nper^n, PV, FV, type)

Example

Problem : Andrew would like to withdraw a fixed amount once a month while he is in law school. How much can he withdraw assuming that he wants to save half the principal? Assume the same interest rate but compounding monthly.

Answer:

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Answer:

$$\text{Let } X = \frac{500((1 + .05)^{21+1} - .05 - 1)}{.05} + 500$$

Example

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Answer:

$$\text{Let } X = \frac{500((1 + .05)^{21+1} - .05 - 1)}{.05} + 500$$

$$A = \frac{.05/12(X/2 - X(1 + .05/12)^{36})}{(1 + .05/12)^{36+1} - (.05/12) - 1}$$

Example

Problem : Andrew would like to withdraw a fixed amount once a month while he is in law school. How much can he withdraw assuming that he wants to save half the principal? Assume the same interest rate but compounding monthly.

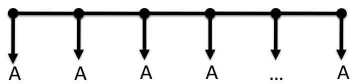
Answer:

A1 = FV(5%, 21, -500, 0, 1) + 500

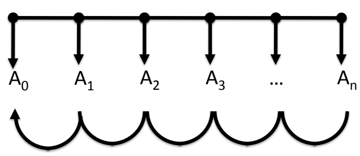
= PMT(5%/12, 36, -A1, A1/2, 1)

= $327

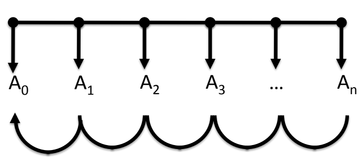
Present Value



Present Value



Present Value



$$PV = \frac{A_0}{(1+i)^0} + \frac{A_1}{(1+i)^1} + \dots + \frac{A_n}{(1+i)^n}$$

Present Value

$$PV = A + \frac{A}{(1+i)} + \ldots + \frac{A}{(1+i)^n}$$

$$= \frac{A}{i}\left[1 - \frac{1}{(1+i)^n}\right]$$

$= PV(\text{rate}^i, \text{nper}^n, A, 0, 1)$

Example

Problem : You want to buy a house for $500,000. You've saved up 20% and your bank is offering to loan you the remaining 80% for 15 years at 7% annual interest compounded monthly. How much interest will you pay in the 8th year?

Answer: See the LoanAmortization spreadsheet.