

**Randomness**

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**Today**

- Basic Ideas
- Applications and Examples
- Monte Carlo Methods

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**Generating Random Numbers**

$0 \leq \text{RAND}() < 1$

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### Example

**Problem:** Simulate the fair flip of a coin. 50% chance heads, 50% chance tails.

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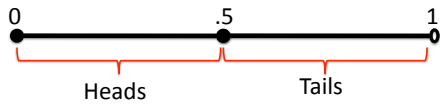
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### Example

**Problem:** Simulate the fair flip of a coin. 50% chance heads, 50% chance tails.



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### Example

**Problem:** Simulate the fair flip of a coin. 50% chance heads, 50% chance tails.

**Solution:**

```
if(rand() < .5, "Heads", "Tails")
```

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### Example

**Problem:** Simulate the flip of a coin. There is a 1/10 of 1% chance that it will land on its side and otherwise equal chances for heads or tails.

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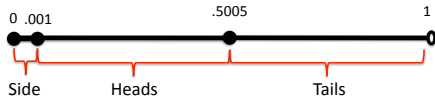
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### Example

**Problem:** Simulate the flip of a coin. There is a 1/10 of 1% chance that it will land on its side and otherwise equal chances for heads or tails.



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### Example

**Problem:** Simulate the flip of a coin. There is a 1/10 of 1% chance that it will land on its side and otherwise equal chances for heads or tails.

**Solution?:**

```
if(rand() < 1%/10,  
   "Side",  
   if(rand() < .5, "Heads", "Tails"))
```

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### Example

**Problem:** Simulate the flip of a coin. There is a 1/10 of 1% chance that it will land on its side and otherwise equal chances for heads or tails.

**Solution:**

```
if(rand() < 1%/10,  
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### Example

**Problem:** Simulate the flip of a coin. There is a 1/10 of 1% chance that it will land on its side and otherwise equal chances for heads or tails.

**Solution?:**

```
A1 = rand()  
if(A1 < 1%/10,  
  "Side",  
  if(A1 < .5, "Heads", "Tails"))
```

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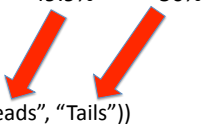
### Example

**Problem:** Simulate the flip of a coin. There is a 1/10 of 1% chance that it will land on its side and otherwise equal chances for heads or tails.

**Solution?:**

```
A1 = rand()  
if(A1 < 1%/10,  
  "Side",  
  if(A1 < .5, "Heads", "Tails"))
```

49.9%      50%



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### Example

**Problem:** Simulate the flip of a coin. There is a 1/10 of 1% chance that it will land on its side and otherwise equal chances for heads or tails.

**Solution?:**

```
A1 = rand()  
if(A1 < 1%/10,  
  "Side",  
  if(A1 < .5, "Heads", "Tails"))
```

49.9%      50%

**WRONG!**

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### Example

**Problem:** Simulate the flip of a coin. There is a 1/10 of 1% chance that it will land on its side and otherwise equal chances for heads or tails.

**Solution:**

```
A1 = rand()  
if(A1 < 1%/10,  
  "Side",  
  if(A1 < .5 + .005, "Heads", "Tails"))
```

49.95%      49.95%

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### Example

**Problem:** Flip a coin 1000 times. Put a formula in A1 that evaluates to the longest streak of the same side.

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### Example

**Problem:** Flip a coin 1000 times. Put a formula in A1 that evaluates to the longest streak of the same side.

**Solution:**

1. B1:B1000 gets `if(rand() < .5, "Heads", "Tails")`
2. C1 gets 1
3. C2 gets `if(B1 = B2, C1 + 1, 1)`
4. Copy C2 to C2:C1000
5. A1 gets `MAX(C1:C1000)`.

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### Generating Random Integers

$$0 \leq \text{rand()} < 1$$

$$0 \leq N * \text{rand()} < N$$

$$\text{INT}(N * \text{rand()}) \text{ in } \{0, \dots, N - 1\}$$

$$\text{RandBetween}(0, N - 1)$$

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### Example

**Problem:** Roll two dice 10 times.

**Solution:**

1. A1 gets `1 + INT(6 * RAND())`
2. Copy A1 to A1:B10

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### Example

**Problem:** There are 36 possible outcomes of the roll of 2 dice. There is 1 way to roll a 2. There are 2 ways to roll a 3, 3 ways to roll a 4, 4 ways to roll a 5, 5 ways to roll a 6 and 6 ways to roll a 7. How close are the expected outcomes when rolling the dice 1000 times?

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### Example

**Problem:** There are 36 possible outcomes ...

**Solution:**

1. A1:B1000 gets  $1 + \text{INT}(6 * \text{RAND}())$
2. C1:C1000 gets  $A_i + B_i$
3. D1:D6 gets 1 thru 6, D7:D11 gets 5 thru 1.
4. E1 gets  $\text{COUNTIF}(C\$1:C\$1000, "=D1")$
5. Copy E1 to E1:E11

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### Monte Carlo Methods

- A method of statistical sampling employed to approximate solutions to quantitative problems.
- Due to John von Neumann and Stanislaw Ulam (1946).

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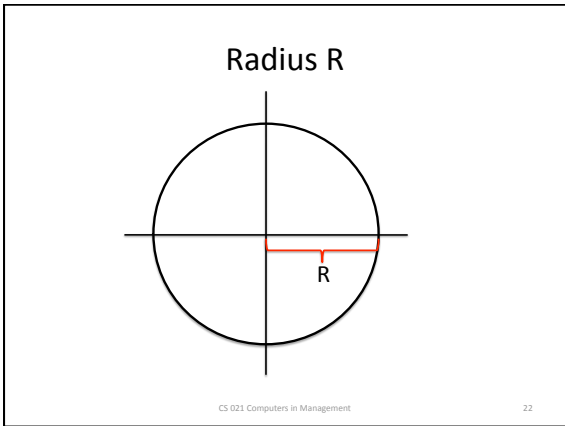
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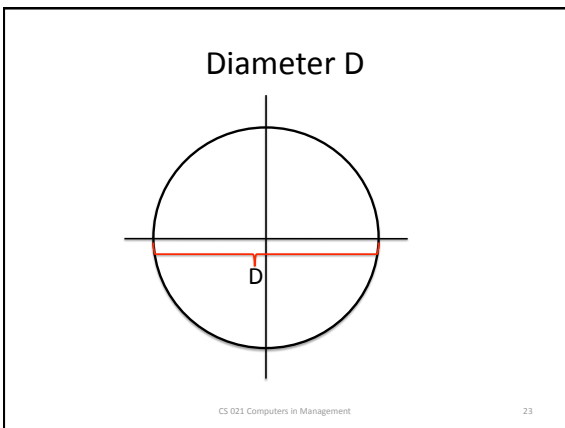
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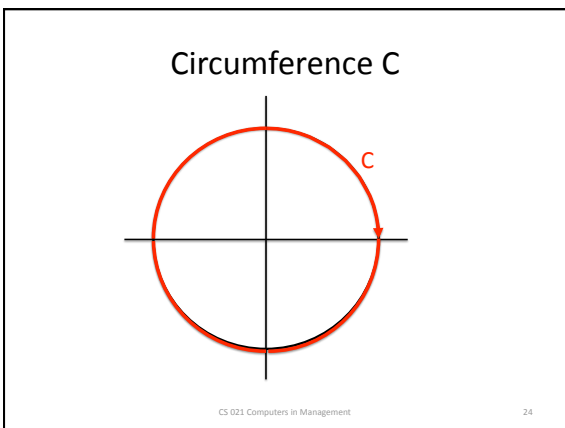
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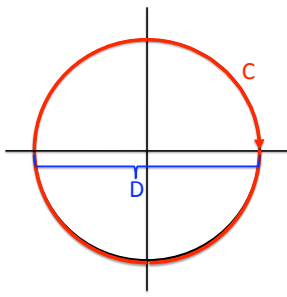
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Circumference/Diameter = 3.14159...



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The diagram shows a circle with a horizontal diameter labeled 'D' and a red circumference labeled 'C'. A vertical line passes through the center of the circle.

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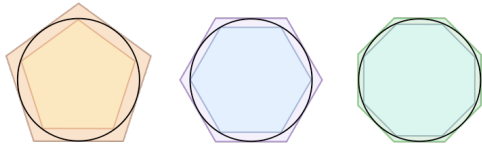
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Archimedes of Syracuse (287 – 212 BC)  
Approximation of  $\pi$



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The diagram shows three polygons approximating a circle: a yellow pentagon, a blue hexagon, and a green octagon. Each polygon is inscribed within a circle.

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James Niles-Joyal '08 : 3,141 Digits



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The image is a portrait of a young man with short brown hair, wearing a blue and white striped polo shirt, standing in front of a brick wall.

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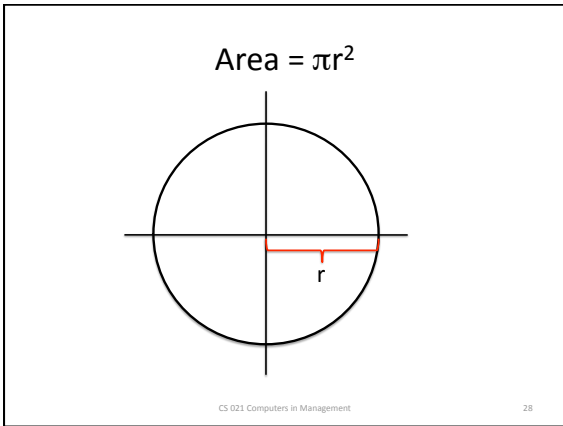
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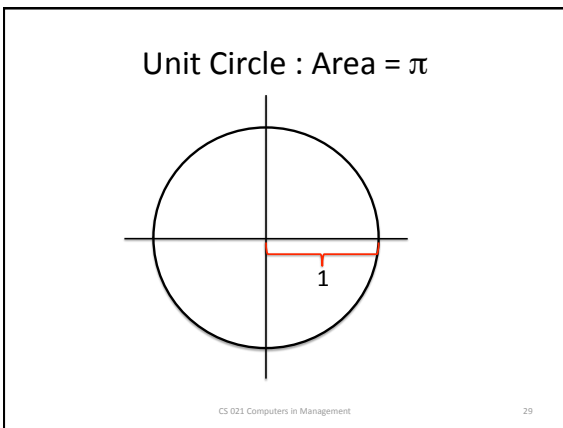
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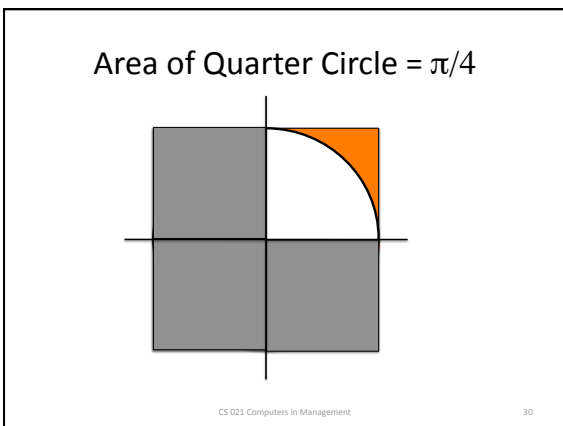
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Area of Inscribing Square = 1

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Randomly Select Points in the Quadrant

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If the points are chosen randomly, then the ratio of points in white to total points in the quadrant should be the same as the ratio  $\pi/4$  to 1.

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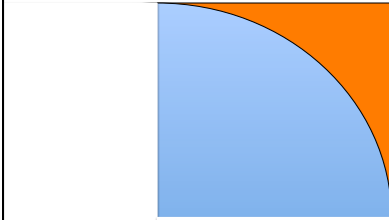
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When is a point in the blue?



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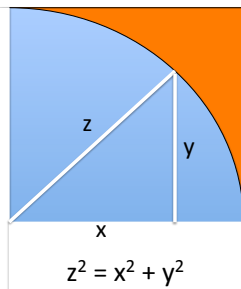
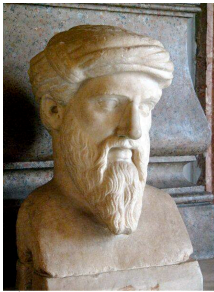
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Pythagoras (570 – 495 BC)



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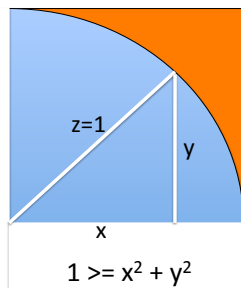
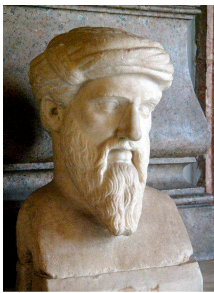
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Pythagoras (570 – 495 BC)



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So ...

- A point  $p = (x, y)$  in the inscribing square is in the quarter circle if  $x^2 + y^2 \leq 1$ .
- We can choose random points in the unit circle by choosing  $x$  and  $y$  randomly.
- I.e., exactly when  $\text{rand}()^2 + \text{rand}()^2 \leq 1$ .

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	A	B	C
1	Monte Carlo Approximation of Pi		
2	Robert Muller		
3	CS 021 Computers in Management		
4			
5	Points	10000	
6			
7	Approxmation	3.1416	
8			
9	<b>x</b>	<b>y</b>	<b>distance</b>
10	0.59912552	0.74215931	0.95380911
11	0.39293882	0.45616591	0.60206997
12	0.10010110	0.00720110	0.00000017

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