Randomness

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CS 021 Computers in Management
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Today

- Basic Ideas
- Applications and Examples
- Monte Carlo Methods

Generating Random Numbers

0 <= RAND() < 1

Example

Problem: Simulate the fair flip of a coin. 50% chance heads, 50% chance tails.

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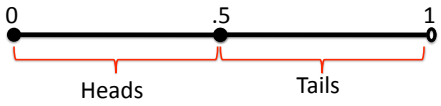

Example

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Solution:

if(rand() < .5, “Heads”, “Tails”)

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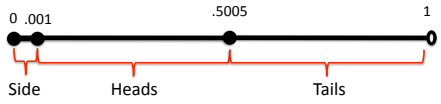

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Problem: Simulate the flip of a coin. There is a 1/10 of 1% chance that it will land on its side and otherwise equal chances for heads or tails.

Solution?:

```
if(rand() < 1%/10,
    "Side",
    if(rand() < .5, "Heads", "Tails"))
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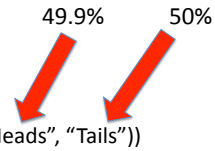

Example

Problem: Simulate the flip of a coin. There is a 1/10 of 1% chance that it will land on its side and otherwise equal chances for heads or tails.

Solution?:

WRONG!

49.9% 50%

```
A1 = rand()
    if(A1 < 1%/10,
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```


Example

Problem: Simulate the flip of a coin. There is a 1/10 of 1% chance that it will land on its side and otherwise equal chances for heads or tails.

Solution:

49.95% 49.95%

```
A1 = rand()
    if(A1 < 1%/10,
        "Side",
        if(A1 < .5 + .005 , "Heads", "Tails"))
```


Example

Problem: Flip a coin 1000 times. Put a forumla in A1 that evaluates to the longest streak of the same side.

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Solution:

1. B1:B1000 gets if(rand() < .5, “Heads”, “Tails”)
2. C1 gets 1
3. C2 gets if(B1 = B2, C1 + 1, 1)
4. Copy C2 to C2:C1000
5. A1 gets MAX(C1:C1000).

Generating Random Integers

0 <= rand() < 1

0 <= N * rand() < N

INT(N * rand()) in {0, …, N – 1}

RandBetween(0, N – 1)

Example

Problem: Roll two dice 10 times.

Solution:

1. A1 gets 1 + INT(6 * RAND())
2. Copy A1 to A1:B10

Example

Problem: There are 36 possible outcomes of the roll of 2 dice. There is 1 way to roll a 2. There are 2 ways to roll a 3, 3 ways to roll a 4, 4 ways to roll a 5, 5 ways to roll a 6 and 6 ways to roll a 7. How close are the expected outcomes when rolling the dice 1000 times?

Example

Problem: There are 36 possible outcomes …

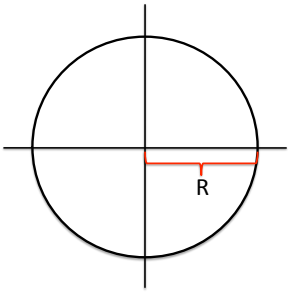
Solution:

1. A1:B1000 gets 1 + INT(6 * RAND())
2. C1:C1000 gets Ai + Bi
3. D1:D6 gets 1 thru 6, D7:D11 gets 5 thru 1.
4. E1 gets COUNTIF(C$1:C$1000, "=D1")
5. Copy E1 to E1:E11

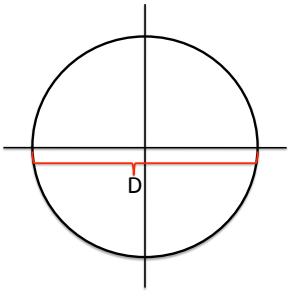
Monte Carlo Methods

- A method of statistical sampling employed to approximate solutions to quantitative problems.
- Due to John von Neumann and Stanislaw Ulam (1946).

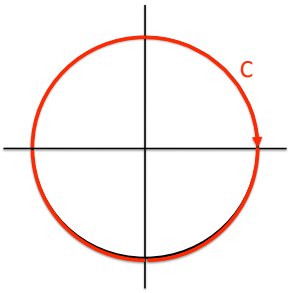
Radius R



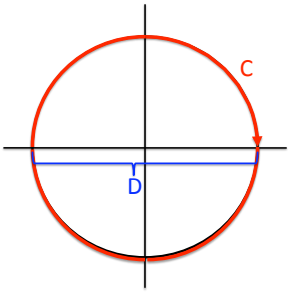
Diameter D



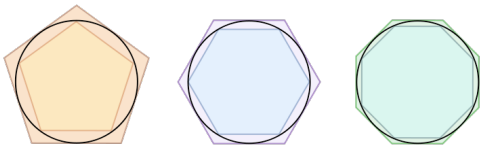
Circumference C



Circumference/Diameter = 3.14159…

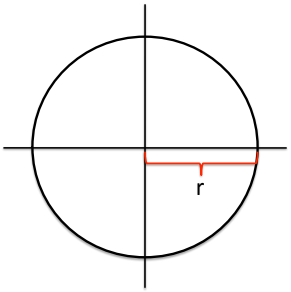


Archimedes of Syracuse (287 – 212 BC)
Approximation of π

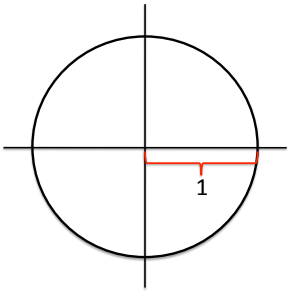


James Niles-Joyal ’08 : 3,141 Digits

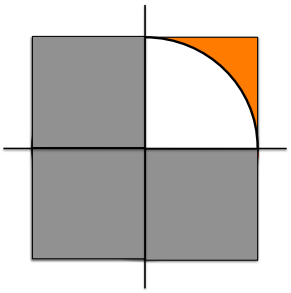
Area = πr^2



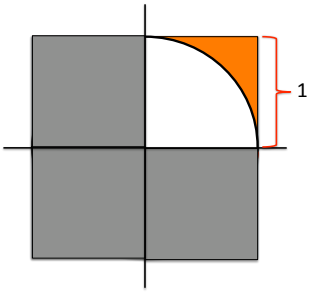
Unit Circle : Area = π



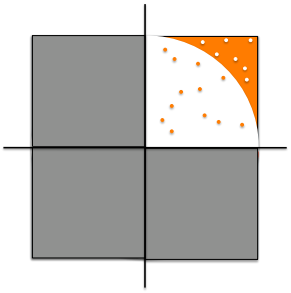
Area of Quarter Circle = $\pi/4$



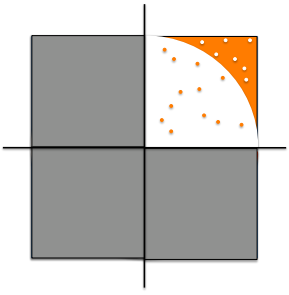
Area of Inscribing Square = 1



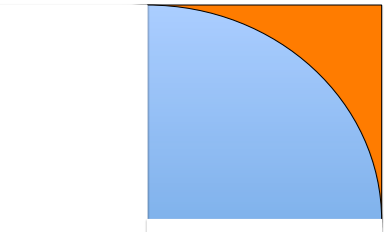
Randomly Select Points in the Quadrant



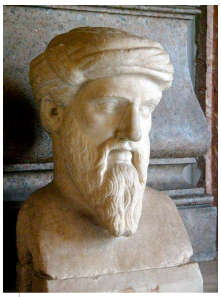
If the points are chosen randomly, then the ratio of points in white to total points in the quadrant should be the same as the ratio $\pi/4$ to 1.

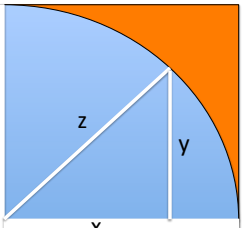


When is a point in the blue?



Pythagoras (570 – 495 BC)





$z^2 = x^2 + y^2$

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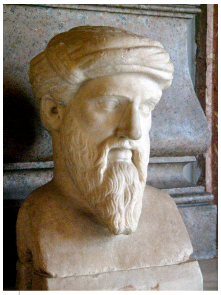

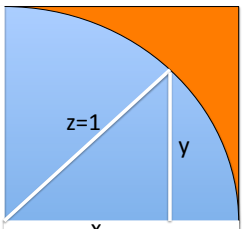

$1 >= x^2 + y^2$

So …

- A point p = (x, y) in the inscribing square is in the quarter circle if $x^2 + y^2 <= 1$.
- We can choose random points in the unit circle by choosing x and y randomly.
- I.e., exactly when rand()^2 + rand()^2 <= 1.

<>	A	B	C
1	Monte Carlo Approximation of Pi		
2	Robert Muller		
3	CS 021 Computers in Management		
4			
5	Points	10000	
6			
7	Approxmation	3.1416	
8			
9	**x**	**y**	**distance**
10	0.59912552	0.74215931	0.95380911
11	0.39293882	0.45616591	0.60206997