

## CS 385 Assignment #1 Solutions

0.5) (4 pts)

If  $|N| = n$ ,  $|P(N)| = 2^n$

Proof by induction:

Base case:  $|N| = 0$ , in this case  $N$  is the empty set, which has only one subset, itself, so  $|P(N)| = 1 = 2^0 = 2^{|N|}$ . So the desired result is true in this case.

Inductive step:

Assume the result holds for some nonnegative integer  $k$ . We must prove: if  $|M| = k+1$ ,  $|P(M)| = 2^{k+1}$ .

We can write  $M = N \cup \{e\}$  for some element  $e$  that is not in  $N$ . Therefore there are two possibilities for each subset of  $M$ : either it does not include  $e$ , in which case it is a subset of  $N$ ; or it has the form  $K \cup \{e\}$  for some subset  $K$  of  $N$ . There are thus exactly twice as many subsets of  $M$  as there are of  $N$ . By the inductive hypothesis,  $|P(N)| = 2^k$ , so  $|P(M)| = 2 \times 2^k = 2^{k+1}$ .

Another way to get the result. Some students gave a simple counting argument to solve this problem: If I want to create a subset of an  $n$ -element set  $N$ , then for each element  $x$  of  $N$ , I need to choose whether to include  $x$  in the subset or not. So I have  $n$  independent choices to make, each with two options. Thus there are  $2^n$  different sequences of choices, and each gives rise to a different subset.

1.0) (3 pts)

if  $a=b$ , then  $a - b = b - b = 0$ . Therefore, we have divided both sides of the equation by 0, which is not a legal operation. (As a simpler example, you cannot conclude  $3=2$  from the fact that  $3 \times 0 = 2 \times 0$ .)

1.1) (3 pts)

Consider the inductive step when  $h = 2$ . The inductive step does prove that, among these two horses, the first is the same color as itself and the second is the same color as itself. However, since these two groups of horses,  $H_1$  and  $H_2$  do not share a horse in common, (the two sets are disjoint), it does not follow that the horses in the two groups must be the same color. (In other words, the only error is when the inductive step is used to pass from 1 horse to 2 horses; for higher values the argument is correct.)

1) (4 pts) There is a link to the program on the website.

3.1) (3 pt)

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Match the first left with the first right. Match the second left with the second right.

OR

Match the second left with the first right. Match the first left with the second right.

3.2) (3 pt)

Theorem 1 has two parts. First, the one-to-one correspondence insures that the number of left parens equals the number of right parens. Second, the fact that the correspondence holds the left paren to the left of the right paren insures that if we split the sequence in two ( $u$  and  $v$ ), for each pair, three things can happen, both sides of the pair are in  $u$ , both sides are in  $v$ , or the left paren is in  $u$  and the right paren is in  $v$ .

Therefore, when looking at the theorem, each paren pair can contribute 1) nothing to  $u$ , 2) both a left and a right to  $u$ , or 3) only a left to  $u$ . (Contributing a right to  $u$  without its left is not possible since that would place the left and right backwards.) Thus, any rights added to  $u$  also add a left, but lefts can be added without adding a right. Therefore, the number of lefts in  $u$  must exceed or equal the number of rights in  $u$  and the second part of the theorem is satisfied.