

PROOF. $p_i(t)$ is the i th coordinate of $\pi \cdot P^{(t)}$, so equals $\sum_{j=1}^n p_{j,i}^{(t)} \pi_j$. Recalling that $\sum_{j=1}^n \pi_j = 1$, we have

$$\begin{aligned}
 |p_i(t) - p_i^*| &= \left| \sum_{j=1}^n p_{j,i}^{(t)} \pi_j - p_i^* \right| \\
 &= \left| \sum_{j=1}^n p_{j,i}^{(t)} \pi_j - \sum_{j=1}^n \pi_j p_i^* \right| \\
 &= \left| \sum_{j=1}^n \pi_j (p_{j,i}^{(t)} - p_i^*) \right| \\
 &\leq \sum_{j=1}^n \pi_j (M_0 - m_0) (1 - 2\epsilon)^t \\
 &\leq (M_0 - m_0) (1 - 2\epsilon)^t.
 \end{aligned}$$