Image Correlation

★ 0 0 0 0 1 = ?
Image Correlation

\[
\begin{bmatrix}
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]
Image Convolution

\[ \begin{array}{ccc}
\otimes & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1 \\
\end{array} \]

= ?
Image Convolution
Image Filtering

\[
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\otimes
= ?
\]
Image Convolution

\[
\begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1
\end{pmatrix}
\times
\begin{array}{c}
\end{array} =
\begin{array}{c}
\end{array}
\]
Image Filtering

\[
\begin{bmatrix}
0 & 0 & 0 \\
0 & 2 & 0 \\
0 & 0 & 0 \\
\end{bmatrix} \times \begin{bmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1 \\
\end{bmatrix} / 9 = ?
\]
Image Filtering

\[
\begin{bmatrix}
0 & 0 & 0 \\
0 & 2 & 0 \\
0 & 0 & 0
\end{bmatrix} \times \begin{bmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1
\end{bmatrix} / 9 =
\]
imfilter is for linear filtering:

```matlab
f = imread('cameraman.tif');
f = im2double(f);
h = ones(3,3)/9;

g = imfilter(f, h, 'corr');  % image correlation
g = imfilter(f, h, 'conv'); % image convolution
```

conv2 can also be used to do the convolution

```matlab
f = conv2(f, h, 'same');
f = conv2(f, h, 'full');
f = conv2(f, h, 'valid');
```
Boundary Issues

- **Full**:
  - $(N+M-1) \times (N+M-1)$

- **Valid**:
  - $N-M+1$

- **Same**:
  - $N \times N$
Special Linear Filters

- **Gaussian filter**

  Continuous form:

  \[
  G_\sigma(x, y) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{(x^2+y^2)}{2\sigma^2}\right)
  \]

  Discrete form:

  \[
  h_{i,j} = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{(i-k-1)^2+(j-k-1)^2}{2\sigma^2}\right)
  \]

  \[
  i = 1 .. 2k + 1, j = 1 .. 2k + 1
  \]
Special Linear Filters

- Gaussian filter

50x50 kernel with sigma 10

50x50 kernel with sigma 20
Gaussian Smoothing

sigma 0.2 1 5 11 21

kernel size

1

5

11

15

19
Special Linear Filters
Kernel Separability

- If \( h(x,y) = h(x)h(y) \), \( h \) is separable.
- For separable kernels, 2D convolution becomes two 1D convolutions:
  - Row 1D convolution followed by the column 1D convolution
- Is Gaussian Filter Separable?
- Is this kernel separable?
  
  \[
  \begin{pmatrix}
  1 & 0 & -1 \\
  2 & 0 & -2 \\
  1 & 0 & -1
  \end{pmatrix}
  \]
- Computational advantage of separable kernel.
Properties of Convolution

Commutative:
\[ f * g = g * f \]

Associative
\[ (f * g) * h = f * (g * h) \]

Superposition
\[ (f + g) * h = f * h + g * h \]

Identity
\[ f * e = f, \text{ where } e \text{ is a unit impulse, e.g. } [0 \ 1 \ 0] \]

Differentiation
\[ \frac{d(f * g)}{dx} = \frac{df}{dx} * g \]
Linear System

Linear: \[ a \cdot f_1 + b \cdot f_2 \Rightarrow a \cdot g_1 + a \cdot g_2 \]
where the response of \( f_1 \) is \( g_1 \)
and the response of \( f_2 \) is \( g_2 \)

Shift invariant:
if \( f \Rightarrow g \), then \( f(n-m) \Rightarrow g(n-m) \)
Nonlinear Filtering

- Neighborhood filtering can be nonlinear

\[ g(x, y) = T_{(u, v) \in N(x, y)}[f(u, v)] \]

- Median Filtering

Mask \[ [1 \ 1 \ 1] \]
Median Filtering in Denoising

Original Image

Add 10% pepper noise
Median Filtering for Denoising

Median filter with 3x3 square structure element
Median Filtering for Denoising

Median filter with 5x5 square structure element
Compared with Gaussian Filtering

Kernel size 5x5 and sigma 3  Kernel size 11x11 and sigma 5