Image Filtering

Matlab

\texttt{imfilter} is for linear filtering:
\begin{verbatim}
f = imread('cameraman.tif');
f = im2double(f);
h = ones(3,3)/9;
g = imfilter(f, h, 'corr');  \texttt{\% image correlation}
g = imfilter(f, h, 'conv'); \texttt{\% image convolution}
\end{verbatim}
\texttt{conv2} can also be used to do the convolution
\begin{verbatim}
f = conv2(f, h, 'same');
f = conv2(f, h, 'full');
f = conv2(f, h, 'valid');
\end{verbatim}

Boundary Issues

Special Linear Filters

- Gaussian filter

Continuous form:
\[ G_\sigma(x, y) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right) \]

Discrete form:
\[ h_{i,j} = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{(i-k-1)^2+(j-k-1)^2}{2\sigma^2}\right) \]
\[ i = 1 \ldots 2k + 1, j = 1 \ldots 2k + 1 \]
Special Linear Filters

- Gaussian filter

Gaussian Smoothing

Kernel Separability

- If $h(x,y) = h(x)h(y)$, $h$ is separable.
- For separable kernels, 2D convolution becomes two 1D convolutions:
  - Row 1D convolution followed by the column 1D convolution
- Is Gaussian Filter Separable?
  - Is this kernel separable?
    - 1 0 -1
    - 2 0 -2
    - 1 0 -1
- Computational advantage of separable kernel.

Properties of Convolution

Commumative:
$$f * g = g * f$$

Associative
$$(f * g) * h = f *(g * h)$$

Superposition
$$(f + g) * h = f * h + g * h$$

Identity
$$f * e = f$$, where $e$ is a unit impulse, e.g. $[0 1 0]$

Differentiation
$$d(f * g)/dx = df/dx * g$$

Linear System

Linear:
$$a f_1 + b f_2 = a g_1 + b g_2$$
where the response of $f_1$ is $g_1$
and the response of $f_2$ is $g_2$

Shift invariant:
$$f \Rightarrow g$$
then $f(n-m) \Rightarrow g(n-m)$
Nonlinear Filtering

- Neighborhood filtering can be nonlinear
  \[ g(x, y) = T_{(u, v) \in N(x, y)} [f(u, v)] \]

- Median Filtering
  
  \[
  \begin{bmatrix}
  1 & 1 & 1 \\
  1 & 2 & 1 \\
  1 & 1 & 1 \\
  \end{bmatrix}
  \] 
  Mask \([1 \ 1 \ 1]\)

Median Filtering in Denoising

Original Image  Add 10% pepper noise

Median Filtering for Denoising

Median filter with 3x3 square structure element

Median Filtering for Denoising

Median filter with 5x5 square structure element

Compared with Gaussian Filtering

Kernel size 5x5 and sigma 3  Kernel size 11x11 and sigma 5