Second Exam
CS 1101 Computer Science I
Spring 2016

Section 03

KEY

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Before reading further, please arrange to have an empty seat on either side of you. Now that you are seated, please write your name **on the back** of this exam.

This is a closed-notes and closed-book exam. Computers, calculators, and books are prohibited.

This is a 26 point exam. Answer all questions in Sections 1 and 2. Answer either 3.1 or 3.2 but not both. Circle the number of the problem that you want graded.

- Partial credit will be given so be sure to show your work.
- Feel free to write helper functions if you need them.
- **Please write neatly.**

Problem	Points	Out Of
1		**6**
2.1		**4**
2.2		**4**
3		**8**
4		**4**
Total		**26**

Section 1 (6 Points Total)

1. (1 Point) Digital computers are "digital" because they use discrete values — digits — to represent things rather than continuous values. Digital computers are actually *binary* digital computers because the discrete values are binary digits (bits) — *everything* in a binary digital computer is represented using the common substrate of bits. For example, the letter **A** is represented as the sequence of 8 bits `0100 0001`. This very same bit sequence is also the representation of the decimal integer `65`, it can also represent 8 booleans, maybe a machine instruction such as `Beq` as well as untold other items.

   In a sentence or two, explain how a digital computer can keep all of this straight. How does it know when to interpret `0100 0001` as a `Beq` rather than `A`?

   Answer: The program counter together with the stored instructions impose an interpretation on the bits.

2. (1 Point) The symbol `<>` is OCaml's not equal operator. Is the following well-defined? If so, what is its value?

```
(match let a = 8 in a <> a with | true -> 3 | false -> 4) + 6
```

   Answer: Yes, answer is 10.

3. (1 Point) Solve for X.

   (a) $C5_{16} = X_4$.

   Answer: We could convert $C5_{16}$ to the equivalent decimal numeral, 197, and then use the iterative algorithm to convert 197 to base 4. However, since 4 is a power of 2, there is an easier way. $C5_{16} = 1100\ 0101_2$. Since $4 = 2^2$, we can regroup the bits in groups of 2: $11\ 00\ 01\ 01 = 3011_4$, so $X = 3011$.

   (b) $C5_{16} = X_8$.

   Answer: As above $C5_{16} = 1100\ 0101_2$. Since $8 = 2^3$, we can regroup the bits in groups of 3: $11\ 000\ 101 = 305_8$, so $X = 305$.

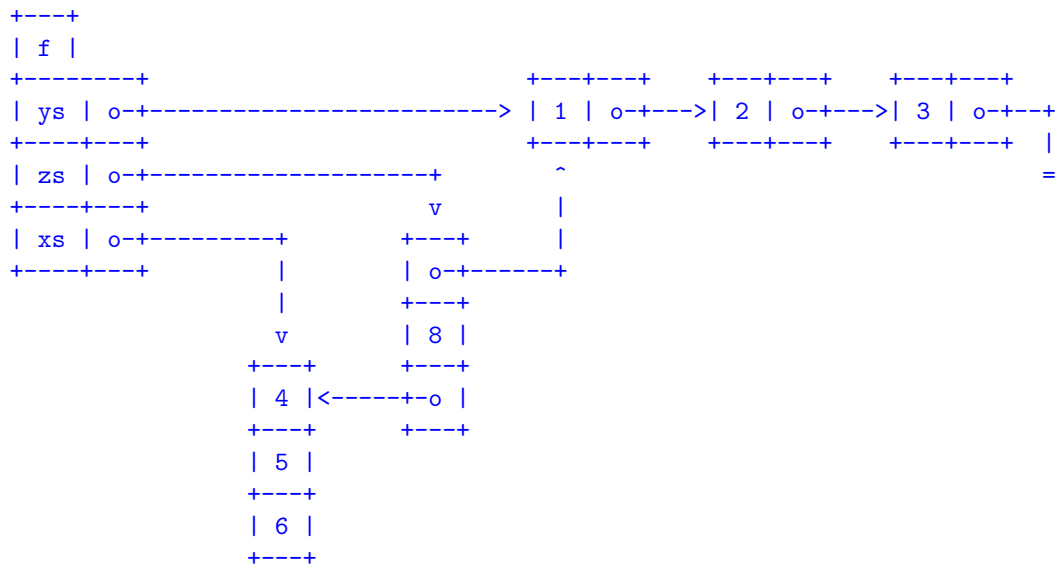
4. (3 Points) Show the state of the stack and heap after (1) has executed but before (2) has executed.

```
let f xs ys =
  let zs = (ys, 8, xs)            (1)
  in
  zs                              (2)

(f [|4; 5; 6|] [1; 2; 3])
```

Stack Heap

Answer:



Section 2 (8 Points Total)

1. (4 Points) Write a function `masterList : (string * 'a list) list -> 'a list` such that a given call `(masterList pairs)` returns a list combining all of the lists in the pairs. For example, the call

   ```
   (masterList [("Haiming", [1.0; 2.0]); ("Alice", [3.0; 4.0])])
   ```

   should evaluate to the list `[1.0; 2.0; 3.0; 4.0]`.

   Answer:

   ```
   let rec masterList pairs =
     match pairs with
     | [] -> []
     | (_, xs)::rest -> xs @ masterList rest
   ```

2. (4 Points) Write a function `arrayDiff : 'a array -> 'a array -> 'a array` such that a given call `(arrayDiff a1 a2)` returns a new array containing all of the elements of `a1` that are not in `a2`. For example, the call `(arrayDiff [|2; 3; 4|] [|3; 5|])` should evaluate to the array `[|2; 4|]`.

   Answer:

   ```
   let arrayDiff a1 a2 =
     let a1' = Array.to_list a1 in
     let a2' = Array.to_list a2 in
     let a3  = List.filter (fun a -> not (List.mem a a2')) a1'
     in
     Array.of_list a3
   ```

Section 3 (8 Points Total)

Do either problem 1 or 2. Do not do both.

1. (8 Points) In statistics, the *population standard deviation* of a set of k numbers n_1, n_2, ..., n_k, is a measure of the *range* of values in the set. Computation of the population standard deviation starts with computing the mean

$$a = (n_1 + n_2 + \ldots + n_k)/k.$$

Then, for each member n_i of the population, we compute a measure of n_i's *deviation* from the mean. The deviation for n_i is $d_i = (n_i - a)^2$. Finally, the population standard deviation is the square root of the average of the d_i:

$$\text{psd} = \sqrt{(d_1 + d_2 + \ldots + d_k)/k}$$

For the purposes of the following three problems, we're going to take the population as a list of floating point numbers. Feel free to use the `sqrt` function in solving these problems. Also feel free to use the function in (a) in your solution to (b) and/or (c) and feel free to use the function in (b) in your solution to (c), even if you weren't able to complete (a) and (b).

(a) (2 Points) Write a function `deviations : float list -> float list` such that a function call `(deviations population)` returns a list of the deviations. For example, the call

```
(deviations [5.0; 3.0; 7.0])
```

should evaluate to the list `[0.0; 4.0; 4.0]` because the average `a = (5.0 +. 3.0 +. 7.0) /. 3.` is `5.0`, and

```
(5.0 -. 5.0) ** 2.0 = 0.0,
(3.0 -. 5.0) ** 2.0 = 4.0 and
(7.0 -. 5.0) ** 2.0 = 4.0.
```

Answer:

```
let average ns =
  let sum = List.fold_left (+.) 0.0 ns
  in
  sum /. (float (List.length ns))

let deviations ns =
  let av = average ns
  in
  List.map (fun n -> (n -. av) ** 2.0) ns
```

(b) (2 Points) Write a function `standardDeviation : float list -> float` such that a call

```
(standardDeviation population)
```

will evaluate to the population standard deviation of `population`. For example, the call

```
(standardDeviation [5.0; 3.0; 7.0])
```

should evaluate to `1.63` because the deviations of `[5.0; 3.0; 7.0]` are `[0.0; 4.0; 4.0]` and `(sqrt ((0.0 +. 4.0 +. 4.0) /. 3.)) = (sqrt (8.0 /. 3.)) = (sqrt 2.666) = 1.63`.

Answer:

```
let standardDeviation ns =
  let devs = deviations ns
  in
  sqrt (average devs)
```

(c) (4 Points) This problem is concerned with computing summary data for student scores. Let's say the input data is a list of pairs:

```
[("Haiming", [80.; 75.]); ("Alice", [99.; 98.]); ("Mark", [70.; 60.])]
```

The desired output is a list of pairs recording how many standard deviations a student's average exam score is from the class average. In this example, the desired result would be

```
[("Haiming" -0.19); ("Alice", 1.27); ("Mark", -1.08)]
```

The overall average of `[80.; 75; 99.; 98.; 70.; 60.]` is

$$(80. + 75. + 99. + 98. + 70. + 60.)/6. = 482./6. = 80.3$$

and the standard deviation is `14.19`. The averages for each of the students (resp.) are

```
(80. + 75.) /. 2.0 = (155. /. 2.0) = 77.5,
(99. + 98.) /. 2.0 = (197. /. 2.0) = 98.5 and
(70. + 60.) /. 2.0 = (130. /. 2.0) = 65.0.
```

So the deviations of the average exam score for each of the students from the overall average are:

```
[77.5 -. 80.3; 98.5 -. 80.3; 65.0 -. 80.3] = [-2.79; 18.20; -15.29]
```

And finally, the number of standard deviations for each student are

```
[-2.79 /. 14.19; 18.20 /. 14.19; -15.29 /. 14.19] = [-0.19; 1.27; -1.08]
```

Write the function `summary : (string * float list) list -> (string * float) list`.

Answer:

```
let summary pairs =
  let ns = masterList pairs in
  let psd = standardDeviation ns in
  let compute (name, scores) =
    let dev = ((average scores) -. (average ns))
    in
    (name, dev /. psd )
  in
  List.map compute pairs
```

2. (8 Points) Write a function `permutations : 'a list -> int -> ('a list) list` such that a call

```
(permutations symbols n)
```

returns a list of all n-length permutations of symbols drawn from `symbols`. For example, the function call `(permutations [0; 1] 3)` should return the 8-element list of length-3 lists:

```
[[0; 0; 0]; [0; 0; 1]; [0; 1; 0]; [0; 1; 1]
 [1; 0; 0]; [1; 0; 1]; [1; 1; 0]; [1; 1; 1]]
```

The ordering of the list elements is unimportant. This is a challenging problem.

Answer:

```
let rec permutations symbols n =
  match n = 0 with
  | true  -> [[]]
  | false ->
      let xs = permutations symbols (n - 1) in
      let ans = List.map (fun sym -> List.map (fun prm -> sym::prm) xs) symbols
                in
                List.fold_left (@) [] ans
```

Section 4 (4 Points)

Let `M` and `N` be non-negative integers and consider a data segment `Data = [M; N; ...]`. Write an SVM program to compute `(isFactor M N)` halting with a `0` in register `R0` if `M` is not a factor of `N` and halting with a `1` in `R0` if `M` is a factor of `N`. Feel free to place any values that you want in the data segment after `M` and `N`. (Yes, this is exactly Part A of problem set 7.)

Answer:

```
data = [M; N]

Lod R0, 0(Zero)   # R0 := M
Lod R1, 1(Zero)   # R1 := N
Cmp R1, R0        # R1 < R0 ?
Blt 2
Sub R1, R1, R0    # R1 := R1 - R0  (N := N - M)
Jmp -4
Cmp R1, Zero      # See R0 is a factor
Beq 2
Li  R0, 0
Hlt
Li  R0, 1
Hlt
```

	Instruction	Meaning
1.	`Lod Rd, offset(Rs)`	Let `base` be the contents of register `Rs` and let `address = base + offset`. Then register `Rd` gets the contents of location `address` in RAM.
2.	`Li  Rd, number`	`Rd := number`.
3.	`Sto Rs, offset(Rt)`	Let `base` be the contents of register `Rt` and let `address = base + offset`. Then RAM location `address` gets `Rs`.
4.	`Mov Rd, Rs`	`Rd := Rs`.
5.	`Add Rd, Rs, Rt`	`Rd := Rs + Rt`.
6.	`Sub Rd, Rs, Rt`	`Rd := Rs - Rt`.
7.	`Mul Rd, Rs, Rt`	`Rd := Rs * Rt`.
8.	`Div Rd, Rs, Rt`	`Rd := Rs / Rt`.
9.	`Cmp Rs, Rt`	`PSW := Rs - Rt`.
10.	`Beq disp`	`PC := PC + disp` if `PSW = 0`.
11.	`Blt disp`	`PC := PC + disp` if `PSW < 0`.
12.	`Bgt disp`	`PC := PC + disp` if `PSW > 0`.
13.	`Jmp disp`	`PC := PC + disp`.
14.	`Jsr disp`	`RA := PC`, `PC := PC + disp`.
15.	`R`	`PC := RA`.
16.	`Hlt`	SVM halts.

Table 1: The SVM Instruction Set. Notation: `Rd` is a destination register, `Rs` and `Rt` are source registers. All of `offset`, `number` and `disp` are integers.