# Test

CS385-Theory of Computation

October 23, 2007

## 1

Consider the language $L_1$ generated by the regular expression

$$a(a \cup b)^*b \cup b(a \cup b)^*a.$$

*(a)* For each of the following four strings, tell whether or not the string is an element of $L_1$:

$$ab, aba, baa, baabb.$$

*(b)* Draw the state-transition diagram of an $NFA$ that recognizes $L_1$. Do this with as few states and arrows as you can.
*(c)* Use the subset construction to find the state-transition diagram of a $DFA$ that recognizes $L_1$. You do not need to include all the subsets of the states in your $NFA$ from part (b), only those that are reachable from the initial state.

## 2

Find a regular expression for the set $L_2$ of strings over $\{a, b, c\}$ that do not contain an occurrence of the segment $ac$. Show your work carefully.

## 3

Consider the operation $\kappa$ on strings over $\{a, b\}$ that replaces each occurrence of $b$ by $ab$. For example, if $w = abba$, then $\kappa(w) = aababa$. Prove that if $L \subseteq \{a, b\}^*$ is a regular language, then the set

$$\{\kappa(w) : w \in L\}$$

is also a regular language. Your argument can use regular expressions or automata, but it must be sufficiently general to apply to all regular languages $L$. At the same time, you can and should *illustrate* your argument with a single example.

## 4

Tell whether each of the following general statements is true or false. If true, give a brief explanation (as part of your explanation, you may cite theorems that were proved either in class or in the textbook). If false, give a counterexample.

*(a)* If $L \subseteq \{a, b\}^*$ is a regular language, then so is the set of all strings in $L$ whose last symbol is $b$.

*(b)* The intersection of two nonregular languages cannot be regular.

*(c)* The complement of a nonregular language cannot be regular.

*(d)* If there is an $DFA$ with 5 states recognizing $L$, then there is a $DFA$ with no more than 5 states recognizing $L^{\mathcal{R}}$.