1
Consider the language $L_1$ generated by the regular expression

$$a(a \cup b)^*b \cup b(a \cup b)^*a.$$ 

(a) For each of the following four strings, tell whether or not the string is an element of $L_1$:

$$ab, aba, baa, baabb.$$ 

(b) Draw the state-transition diagram of an NFA that recognizes $L_1$. Do this with as few states and arrows as you can.

(c) Use the subset construction to find the state-transition diagram of a DFA that recognizes $L_1$. You do not need to include all the subsets of the states in your NFA from part (b), only those that are reachable from the initial state.

2
Find a regular expression for the set $L_2$ of strings over $\{a, b, c\}$ that do not contain an occurrence of the segment $ac$. Show your work carefully.

3
Consider the operation $\kappa$ on strings over $\{a, b\}$ that replaces each occurrence of $b$ by $ab$. For example, if $w = abba$, then $\kappa(w) = aababa$. Prove that if $L \subseteq \{a, b\}^*$ is a regular language, then the set

$$\{\kappa(w) : w \in L\}$$

is also a regular language. Your argument can use regular expressions or automata, but it must be sufficiently general to apply to all regular languages $L$. At the same time, you can and should illustrate your argument with a single example.
Tell whether each of the following general statements is true or false. If true, give a brief explanation (as part of your explanation, you may cite theorems that were proved either in class or in the textbook). If false, give a counterexample.

(a) If $L \subseteq \{a, b\}^*$ is a regular language, then so is the set of all strings in $L$ whose last symbol is $b$.

(b) The intersection of two nonregular languages cannot be regular.

(c) The complement of a nonregular language cannot be regular.

(d) If there is an $DFA$ with 5 states recognizing $L$, then there is a $DFA$ with no more than 5 states recognizing $L^R$. 