CSCI2243-Assignment 7

Assigned October 31, due November 8 at 11 PM

A mathematical induction workout. Turn in solutions to the textbook problems $1(a \wedge (d \vee e))$,5, 12(a,b), 14, 18 of Chapter 6. 12(d) will get extra credit.

NOTES: If you do 1(e) you don't have to turn in the solution to 1(d), but will get slightly more credit for (e). Problem 5 is different from our other induction problems using summations, because when we pass from n to $n+1$, we do not just add a single term to the sum, but 2^n new terms (this is a hint!) Problems 12(a,b) pretty much follow the template of the proof in the book and together provide a fairly good estimate of the growth of the Fibonacci sequence, namely

$$1.5^n < F_n < 1.7^n$$

for all but a few small values of n. You can marvel at the weird and wonderful formula in 12(d) (and wonder how it is possible that the formula only gives integer values), or try to prove it for extra credit.

Additional Extra-Credit Problem. Change the rules of the unstacking game described in the text so that on each move you receive the *sum* of the sizes of the two stacks instead of the product. So for example, with an initial stack size of 3, there is only one way to proceed, and it gets you 3 points on the first turn, and 2 points on the second, for a total of 5. However (unlike the multiplicative version) with an initial stack of 4, the two strategies give different results.

Find the optimal strategy, along with a formula for the maximum number of points you can get with an initial stack size of n, and prove that your answer is correct (that is, prove both that the formula is correct and that it gives the maximum number of points). It's easy to guess what the best strategy is after a few trials, but not so easy to show that this is correct!