What the well-dressed homework assignment is wearing

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Here the solution was prepared using LaTeX. The markup language is fairly complex, especially in the construction of the table, but you can get very far just by imitating the examples.

1 The problem

Find all the satisfying assignments of the formula

$$(p \oplus q) \to (p \wedge r).$$

2 Solution 1: A truth table

Below is the truth table. Notice that five rows give the value **T** in the last column. These rows correspond to the five satisfying assignments with p and q both true, p and r both true, and p and q both false.

p	q	r	$\phi = p \oplus q$	$\psi = p \wedge r$	$\phi \to \psi$
T	T	T	F	T	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	T	F	F
F	T	T	T	F	F
F	T	F	T	F	F
F	F	T	F	F	T
F	F	F	F	F	T

3 Solution 2: Using algebra.

We only have to expand the formula, replacing the connectives $\oplus$ and $\to$ by equivalent formulations using just $\wedge, \vee$ and $\neg$. This gives us a DNF formula equivalent to the original:

$$\begin{aligned}(p \oplus q) \to (p \wedge r) &\equiv \neg(p \oplus q) \vee (p \wedge r) \\ &\equiv (p \wedge q) \vee (\neg p \wedge \neg q) \vee (p \wedge r).\end{aligned}$$

The DNF has only three disjuncts, but each one represents two different satisfying assignments. We get the same result as with the truth table: Any assignment that has both p, q true, both p, q false, or both p, r true is satisfying; any other is not satisfying.