Assignment 5: Part 1

CSCI2244-Randomness and Computation
Part 1 due Friday, March 15 at 11:59

The first part of the assignment consists of some routine pencil-and-paper problems about conditional probabilities and Bayes's Theorem. When you write up your solution, make sure that you carefully identify the underlying events and the conditional probabilities that are involved. (Most of these problems are lifted from the textbook by Grinstead and Snell.)

The second part of the assignment is a kind of mini-project, to create a Naïve Bayes Classifier to determine the source of some text in English. This will be posted separately, and due a week later.

1 Written Problems

1. (a) You roll a die twice. The first die comes up 4. What is the probability that the sum of the two rolls is at least 7?

   (b) You roll a die twice. At least one of the rolls is 4. What is the probability that the sum of the two rolls is at least 7?

2. (a) There are three drawers in a cabinet. One drawer contains two gold balls, one two silver balls, and the third drawer a gold ball and a silver ball. You pick a drawer at random, reach in without looking, and pull out a ball. It is gold. What is the probability that you picked the drawer with two gold balls?

   (b)A high school senior applies to both Boston College and Northeastern University. She estimates that the probability of her being accepted by Northeastern is 0.5, and by BC 0.3. She also estimates her probability of being accepted by both schools at 0.18. The first news she receives is a letter of admission to Northeastern, and based on this she upgrades her estimate of the likelihood of being admitted to BC. What is the new estimate? What would her estimate be if instead she had been rejected by Northeastern? Are the events 'admitted to BC' and 'admitted to Northeastern' independent events?

3. (a) One coin in a collection of 65 coins has two heads. The rest are fair. You choose a coin at random from the collection and toss it 6 times. It comes up heads every time. What is the probability that it is the two-headed coin? (Think of this like our diagnostic drug testing example: The two-headed coin is a user; the diagnostic test is flipping the coin and seeing if it comes up heads. Note that the 'false negative' rate in this example is zero, because if the test gives a negative result—that is, if the coin ever comes up tails—then we know it's not the two-headed coin.)

   (b) Here is a more nuanced version of the same problem. Imagine that you have a job as a coin-tester. You know that the coins you are given to test are either unfair coins with heads probability $\frac{1}{4}$, unfair coins with heads probability $\frac{3}{4}$, and fair coins. You are given a coin to test. At the outset, you have no reason to believe that the coin is more likely to be of one type than of either of the other types, so you assign a probability of $\frac{1}{3}$ to each of these alternatives. You then flip the coin seven times and get 4 heads and 3 tails. As a result of this (and Bayes's Theorem), you re-calibrate your beliefs and assign new probabilities to each of the three alternatives. What are they? What if you made 70 tosses and got 40 heads?