CS127-First Exam

October 28, 2008

1 Number Representation and Roundoff Error

(a) Show the computation of $(1 + 1/3) - 1$ using decimal arithmetic with three digits of precision. That is, each nonzero number is represented in the form

$$\pm a.bc \times 10^n,$$

where a, b, c are decimal digits, and n is an integer. The result of each operation is rounded off to the nearest value that can be represented exactly in this form.

Solution.

[redacted]

(b) Consider the following sequence of MATLAB commands, along with the results that were printed.

```
>> x=1/1000
x =

    1.000000000000000e-003

>> x==(1+x)-1
ans =
```

```
     0
>> x=1/1024
x =

    9.765625000000000e-004
>> x==(1+x)-1
ans =

     1
```

Of course, for any real number x, $(1+x)-1$ and x are equal. Why did MATLAB say that they are equal in one instance but unequal in another?

Solution.

[redacted]

2 Branching

A student creates an M-file whose contents are shown below.

```
function testif(n)

if(n<7 & n>3)
    if(mod(n,2)=0)
        printf('yes\n');
    else
        printf('no\n');
else
    printf('maybe\n');
end
```

When she types

```
testif(10)
```

at the command prompt, MATLAB responds with an error message:

```
??? Error: File: testif.m Line: 4 Column: 16
The expression to the left of the equals sign is not a valid target for an assignment.
```

She quickly locates the source of the error, corrects it, and tries again. This time, MATLAB responds

```
??? Error: File: testif.m Line: 8 Column: 1
Illegal use of reserved keyword "else".
```

After some, reflection, she discovers the reason for the error, runs the program successfully, and gets the output `Maybe`.
(a) Correct the errors in the program. You don't have to rewrite everything; just tell where the errors are and what you need to put in to correct them.

Solution. [redacted]

(b) Find values for the input parameter `n` that lead to outputs `Yes` and `No`.

Solution. [redacted]

3 Solving Nonlinear Equations

Based on a true story. A student, attempting to implement the secant method, stumbles on a new algorithm for solving equations of the form $f(x) = 0$. Start with an *anchor* point v and a guess x_0 and at each step, set x_{n+1} to be the point where the line joining $(x_n, f(x_n))$ to $(v, f(v))$ crosses the x-axis.

Suppose we use this to solve $x^2 - 1 = 0$ with anchor $v = 2$ and initial guess $x_0 = 0$. In this case the iteration rule can be rewritten

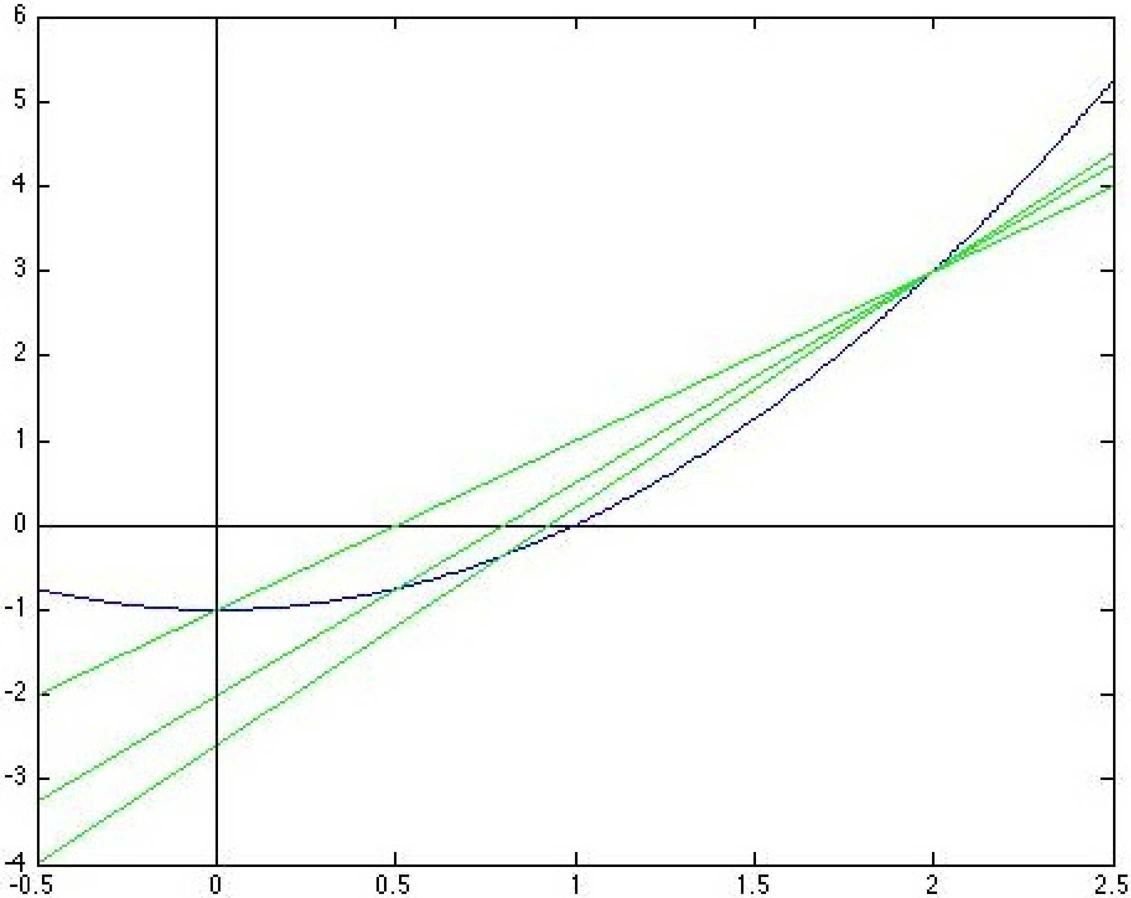
$$x_{n+1} = \frac{2x_n + 1}{x_n + 2}.$$

The first few steps are shown in the plot below.
(a) Compute x_1, x_2, and x_3.

Solution. [redacted]

(b) Let ϵ_n denote the absolute error in the n^{th} iterate. Verify that

$$\epsilon_{n+1} = \frac{\epsilon_n}{3 - \epsilon_n}.$$



6
5
4
3
2
1
0
-1
-2
-3
-4
-0.5
0
0.5
1
1.5
2
2.5

Solution.

$$
\begin{aligned}
\epsilon_{n+1} &= 1 - x_{n+1} \\
&= 1 - \frac{2x_n + 1}{x_n + 2} \\
&= \frac{1 - x_n}{x_n + 2} \\
&= \frac{1 - x_n}{3 - (1 - x_n)} \\
&= \frac{\epsilon_n}{3 - \epsilon_n}
\end{aligned}
$$

What does this say about the rate of convergence? (Linear? Quadratic? Something else?) Is this consistent with your answer in (a)?

Solution.

[redacted]

(c) The following MATLAB function was used to implement this solution method for the equation $x^2 - 1 = 0$ with anchor 2. The function allows you to vary the initial guess x_0, and is supposed to return the computed value of the root (presumably 1) as well as the number of iterations required to converge.

```
function [sol,numit]=funnysecant(guess)

numit=0;
x=guess;
while(x^2-1~=0)
```

```
    x=(2*x+1)/(x+2);
end
numit=numit+1;
sol=x;
```

This function contains several errors which cause it to (i) never terminate for certain initial guesses (even guesses for which the method actually does converge) and (ii) to never report the correct number of iterations. Describe these errors and correct them.

Solution.
[redacted]

[redacted]

[redacted]

4 Polynomial Interpolation

(a) Compute the second degree interpolating polynomial for the function $y = \sin \pi x$ at the points $x = 0, \frac{1}{2}, 1$. Your answer should be in the form $ax^2 + bx + c$.

Solution.
[redacted]

[redacted]

(b) Suppose you want to know the maximum absolute error that results from approximating $sin\pi x$ on the interval from 0 to 1 using this polynomial. Describe how to go about finding this. (You dont have to give the numerical answer.)

Solution. [redacted]

[redacted]