P. Clote

Math Primer
\[ [\mathcal{A}]_{\mathcal{I}} \mathcal{P} = 1 = [\mathcal{V} - \mathcal{U}]_{\mathcal{I}} \mathcal{P} \]

2. 
\[ \mathcal{I} = [\mathcal{U}]_{\mathcal{I}} \mathcal{P} = 0, \quad [\emptyset]_{\mathcal{I}} \mathcal{P} = \mathcal{I} \]

Laws for probability

\( (\varepsilon)_{\mathcal{I}} \mathcal{P}^{\mathcal{V} \in \mathcal{A}} \subseteq = [\mathcal{A}]_{\mathcal{I}} \mathcal{P} \) satisfies \([\mathcal{I}, 0, 1] \leftarrow (\mathcal{U})_{\mathcal{I}} \mathcal{P} \)

• Elementary probability function \( p \) with \([\mathcal{I}, 0, 1] \leftarrow \mathcal{U} \): non-empty set of mutually exclusive

• Elementary events \( \mathcal{E} \) non-empty sample space \( \mathcal{U} \)

\( n/(\mathcal{A})_{\mathcal{I}} \mathcal{P} \) in trials \( n \)

Probability theory
\[
\frac{[V]_{\uparrow}d}{[B]_{\uparrow}d [B \mid V]_{\uparrow}d} = [V \mid B]_{\uparrow}d
\]

\text{Bayes' rule} \quad \bullet

\text{Assuming } B_1, \ldots, B_n \text{ mutually exclusive and exhaustive}

\[
\sum_{i=1}^{n} [B_i]_{\uparrow}d [B_i \mid V]_{\uparrow}d = [V]_{\uparrow}d
\]

\text{Total probability formula} \quad \bullet

\text{Conditional probability: } P(\cdot|\cdot)

\[
[B \mid V]_{\uparrow}d = [V]_{\uparrow}d
\]

\forall \text{ independent}, \forall \text{ } [B]_{\uparrow}d [V]_{\uparrow}d = [B \mid V]_{\uparrow}d

\text{The probability of event } \emptyset \text{ is then } P(\emptyset) = [\{\emptyset\}]_{\uparrow}d

\text{For all } \forall A, \forall B \subseteq \emptyset, \forall [B \cup A]_{\uparrow}d - [B]_{\uparrow}d + [A]_{\uparrow}d = [B \cap A]_{\uparrow}d \quad \text{3.}
measurable function.

Continuous random variable: $X : \mathbb{R} \rightarrow \mathbb{R}$ and $X$ and $\mathbb{R}$, where $\mathbb{R}$.

Countable infinite.

Discrete random variable: $X : \mathbb{R} \rightarrow \mathbb{R}$, where $\mathbb{R}$ finite or countable.

Application later for hidden Markov models.

$$
\frac{\mathbb{E}_X[\mathbb{E}_B[A|B]]}{\mathbb{E}_X[\mathbb{E}_B[A|B]]} = \mathbb{E}_X[\mathbb{E}_B[A|B]]
$$

mutually exclusive and exhaustive. Then

**Generalized Bayes’ Rule**: Suppose hypotheses $B_1, \ldots, B_n$ are


tence $\frac{\mathbb{P}[A]}{\mathbb{P}[B|A]} = \mathbb{P}[B|A]$ and

justification: None
\[ I = x \rho (x) \int_{\infty}^{p} \text{d} \]

Thus

\[ x \rho (x) \int_{q}^{p} \text{d} = \left[ q \geq (\alpha) X \geq v \mid U \ni \alpha \right] \text{d} = [q \geq X \geq v] \text{d} \]

must satisfy

\[ \mathcal{R} \leftarrow \mathcal{R} : x \text{d} \]

(\text{fpd}) \text{ (probability density function)}

In continuous case, \[ I = \left[ x = X \right] \text{d} ^{\infty} = \int_{\infty}^{\infty} \text{d} \]

Thus

\[ x = (\alpha) X \mid U \ni \alpha \]

\[ (\alpha) \text{d} = \left[ \{ x = (\alpha) X \mid U \ni \alpha \} \right] \text{d} \]

For discrete case, \[ [x = X] \text{d} \]
In continuous case, expectation of $X$
\[ E[X] = \int_{-\infty}^{\infty} x \, p_X(x) \, dx. \]

In discrete case, expectation of $X$
\[ E[X] = \sum_{i=-\infty}^{\infty} i \cdot \Pr[X=i]. \]

Hence
\[ P_a \leq X \leq b] = \Phi_X(b) - \Phi_X(a) = \int_{a}^{b} p_X(x) \, dx. \]
\[ x = X \mathcal{L} \sum_{-\infty}^{\infty} x = [\mathbb{E}X] \mathcal{L} \]

In discrete case

mean square or second moment of \( X \) of \( X \)

\[
[\mathbb{E}X] \mathcal{L} \cdot [\mathbb{E}^2X] \mathcal{L} = [\mathbb{E}X^2] \mathcal{L}
\]

If \( \mathbb{E}X^2 \) and \( \mathbb{E}X^1 \) are independent then

\[
[\mathbb{E}^2X] \mathcal{L} \geq [\mathbb{E}X^1] \mathcal{L} \quad \text{implies} \quad [\mathbb{E}^2X] \mathcal{L} \geq [\mathbb{E}X^1] \mathcal{L}
\]

For any \( q \in \mathbb{R} \),

\[
[\lambda] \mathcal{L} q + [X] \mathcal{L} q = [\lambda q + X^q] \mathcal{L}
\]

Expectation is linear.
\[z^\mu - X] = [X] \Lambda\]

where, \( [X] \) is second moment of \( [X] \Lambda \) variance.

\[ \int_{-\infty}^{\infty} x p(x) \text{d}x = [\mu X] \]

- In continuous case

\[ [x = X] \int_{-\infty}^{\infty} \text{d}x \]

- In discrete case

\[ \int_{-\infty}^{\infty} x p(x) \text{d}x = [\mu X] \]

- In continuous case
\[ [\lambda] \Lambda + [X] \Lambda = [\lambda + X] \Lambda \]

If \( \lambda \) and \( X \) are independent then variance is additive:

\[ [X] \Lambda \sigma^2 = [X^2] \Lambda \]

\[ [X] \Lambda^\wedge = (X)^0 \]

\[ n \hat{\sigma} - [n \hat{X}] \hat{\theta} = \]

\[ n \hat{\sigma} + n \hat{\sigma} - [n \hat{X}] \hat{\theta} = \]

\[ n \hat{\sigma} + [X] \hat{\theta} n \hat{\sigma} - [n \hat{X}] \hat{\theta} = \]
\[ q \geq x \geq a \quad \frac{0}{1} \quad \left\{ \begin{array}{l}
\end{array} \right\} = (x) x d
\]

where density function

\[
\frac{v - q}{a - p} = \frac{v - q}{xp} \int_p^\infty = [p > X > a] \cdot d
\]

In continuous case, \( X \) is distributed uniformly \([q, a]\). •

\[
\left| \mathcal{U} \right| = \left[ \omega = X \right] \cdot d
\]

In discrete case •

\( \cup \)

Uniform distribution

Probability Distributions
This is used later in the algorithm to generate normally distributed random reals.

\[
\frac{\frac{1}{2}}{\frac{p^2 + qpq - q}{q + qpq + q}} = \frac{\frac{q}{q + qpq + q} - \frac{\epsilon}{p + qpq + q}}{
= \left( \frac{q}{q + q} \right) - \frac{(p - q)\epsilon}{p - q} = \frac{(p - q)q}{q + q}
\int_x^p \frac{q}{q} \quad \text{is} \ [X]_{\Lambda}
\]

and

\[
\frac{\frac{q}{q + q} = \frac{(p - q)q}{q + q}}{x} = \int_x^p \frac{q}{q} \quad \text{is} \ [X]_{\tilde{A}}
\]

In continuous case, •
\[
x p \frac{\nu \gamma / \zeta}{\zeta / \zeta} \left( x - \varnothing x - \infty \right) = 0
\]

By integration by parts,

\[
0 = x p \frac{\nu \gamma / \zeta}{\zeta / \zeta} \left( x - \varnothing x - \infty \right) \int = [X] \Phi
\]

By symmetry.

\[
\frac{\nu \gamma / \zeta}{\zeta / \zeta} = (x) x d
\]

Probability density function (bell-shaped curve centered at 0).

Normal (Gaussian distribution) with mean 0 and variance 1.
Given as \( n \rightarrow \infty \).

In biology, data often assumed normally distributed, so values \( \mu \) and \( \sigma \).

Within \( 1 \) resp. \( 2 \) standard deviations \( \sigma \) of \( \mu \), roughly \( 68\% \) resp. \( 95\% \) of area under the curve of \( f(x) \) has.

\[
\frac{-\sigma \sqrt{2\pi} e^{-(\frac{x-\mu}{\sigma})^2}}{\sqrt{2\pi} e^{-(\frac{x-\mu}{\sigma})^2}} = (x)dx
\]

Probability density function of normal distribution with mean \( \mu \) and variance \( \sigma^2 \) has.

\[
I = [X] \Lambda \text{ so } \frac{1}{\sqrt{2\pi} e^{-(\frac{x-\mu}{\sigma})^2}} = e^{0} = e^{0}
\]

\[
x e^{\frac{\nu Z/\zeta}{Z}} \int_{-\infty}^{\infty} f(x)dx = e^{0} \int_{-\infty}^{\infty} e^{\frac{\nu Z/\zeta}{Z}}dx
\]
\[ \mathbb{E}[X] = \frac{1}{n} \sum_{i=1}^{n} x_i \]

For each \( X \), \( n \), \( \bar{X} \) are independent, uniformly distributed on \([0, 1]\).

Approximation of normal distribution: Let \( X_1, \ldots, X_n \) be independent, normally distributed with mean \( \mu \) and variance \( \sigma^2 \). Let \( X \) be independent, identically distributed \( \mathcal{N}(\mu, \sigma^2) \). Let \( \bar{X} \) be independent, identically distributed \( \mathcal{N}(\mu, \sigma^2) \). Let \( \bar{X} \) be independent, identically distributed \( \mathcal{N}(\mu, \sigma^2) \). Let \( \bar{X} \) be independent, identically distributed \( \mathcal{N}(\mu, \sigma^2) \). Let \( \bar{X} \) be independent, identically distributed \( \mathcal{N}(\mu, \sigma^2) \). Let \( \bar{X} \) be independent, identically distributed \( \mathcal{N}(\mu, \sigma^2) \). Let \( \bar{X} \) be independent, identically distributed \( \mathcal{N}(\mu, \sigma^2) \). Let \( \bar{X} \) be independent, identically distributed \( \mathcal{N}(\mu, \sigma^2) \). Let \( \bar{X} \) be independent, identically distributed \( \mathcal{N}(\mu, \sigma^2) \). Let \( \bar{X} \) be independent, identically distributed \( \mathcal{N}(\mu, \sigma^2) \). Let \( \bar{X} \) be independent, identically distributed \( \mathcal{N}(\mu, \sigma^2) \). Let \( \bar{X} \) be independent, identically distributed \( \mathcal{N}(\mu, \sigma^2) \). Let \( \bar{X} \) be independent, identically distributed \( \mathcal{N}(\mu, \sigma^2) \). Let \( \bar{X} \) be independent, identically distributed \( \mathcal{N}(\mu, \sigma^2) \). Let \( \bar{X} \) be independent, identically distributed \( \mathcal{N}(\mu, \sigma^2) \). Let \( \bar{X} \) be independent, identically distributed \( \mathcal{N}(\mu, \sigma^2) \). Let \( \bar{X} \) be independent, identically distributed \( \mathcal{N}(\mu, \sigma^2) \). Let \( \bar{X} \) be independent, identically distributed \( \mathcal{N}(\mu, \sigma^2) \). Let \( \bar{X} \) be independent, identically distributed \( \mathcal{N}(\mu, \sigma^2) \). Let \( \bar{X} \) be independent, identically distribut
\[
\frac{[uS] \Lambda}{[uS] \mathcal{A} - uS} \mathcal{A} \phi + \eta = [uS] \mathcal{A} \phi + \eta = [uS \phi + \eta] \mathcal{A}
\]

By linearity of expectation.

\[
[X] \mathcal{A} \phi + \eta = [X \phi + \eta] \mathcal{A}
\]

By Central Limit Theorem.

\[
\frac{\sqrt{1/2}}{\sqrt{u}/2 - \langle X \rangle_{u} \mathcal{A}}
\]

approximately normal with mean 0 and variance 1.

\[
\frac{1/2}{\sqrt{u}/2 - \langle X \rangle_{u} \mathcal{A}}
\]

By Central Limit Theorem.

\[
1/12 = [X] \Lambda
\]

and
Algorithm

\[ z^0 = 1 \cdot z^0 = \]
\[ \left[ \frac{[uS] \Lambda}{[uS] H - u_S} \right] \Lambda z^0 = \]
\[ \left[ \frac{[uS] \Lambda}{[uS] H - u_S} \right] \Lambda z^0 = \left[ uS \phi + \eta \right] \Lambda \]

\[ [X] \Lambda z^0 = [X \phi + \eta] \Lambda \]

\[ \eta = 0 \cdot \phi + \eta = \]
\[ \left[ \frac{u \Lambda \phi}{\eta u - u_S} \right] H \phi + \eta = \]
\[ uX + \cdots + 1X = X \]

is

For \( n \) independent trials, the number \( X \) of successes in \( n \) trials

success is 1 and failure is 0.

\[
\{ 1, 0 \} \leftarrow \mathcal{U}: X
\]

Bernoulli trial is experiment with probability of success

**Binomial Distribution**

{mean,sqrt(variance)*exp(-N/2)/(sqrt(N)*sqrt(1.2, 0))

\[
\text{return for } i=0; i<N; i++ \text{ \{ double } x = + \text{ rand() rand_MAX}/() \text{ } \}
\]

\text{int \_\_double x=0;}

\text{const int N = 20; }
\[(d - 1)du = [X] \Lambda \]

and

\[du = [X] \mathcal{A}\]

so by additivity of expectation and variance

\[(d - 1)d = \mathcal{C}d - d = \mathcal{C}^{\mathcal{A}} \mathcal{A} - \mathcal{C}^{\mathcal{A}} \mathcal{A} = [\mathcal{A}] \Lambda \]

and \(d = [\mathcal{A}] \mathcal{A}\) won

\[\cdot \mathcal{y} \cdot u (d - 1) \mathcal{y} \mathcal{d} (\mathcal{y} \quad \mathcal{u}) = (\mathcal{y} \cdot u) q = [\mathcal{y} = X] \mathcal{d}\]

where each \(\mathcal{y} \cdot \mathcal{A}\) is an independent Bernoulli r.v.
\[
\sum_{n=0}^{\infty} P^n x^n \cdot e^{-x} = x^n
\]

\[\text{Nth term in Taylor expansion of}
\]

\[\text{for } \lambda \in \mathbb{N}.
\]

\[
\lambda \cdot e^{-\lambda} \int_{\{\gamma = \lambda\}} d\lambda = [\gamma = X] d\lambda
\]

\[\text{X has Poisson distribution with parameter } \lambda.
\]

\[\text{Poisson Distribution}
\]

\[\text{number of successes}.
\]

\[\text{which is the Poisson distribution with mean np expected}
\]
\[ Y = \chi \partial Y - \partial \chi = \] 
\[ (\chi \partial) \frac{\chi p}{p} \chi - \partial \chi = \] 
\[ \left( \frac{i \gamma}{\gamma T} \sum_{0}^{\infty} \right) \frac{\chi p}{p} \chi - \partial \chi = \] 
\[ \chi - \partial \frac{i \gamma}{\gamma T} \sum_{0}^{\infty} = [X] H \] 
\[ [X] \Lambda = Y = [X] H \quad \text{Note} \] 
\[ \exists \] 
\[ \exists \text{OS} \left( \frac{i \gamma}{\gamma x} \right) \]
\[ \mathcal{Y} = \mathcal{Z} \mathcal{X} - \mathcal{Z} \mathcal{X} + \mathcal{Y} = \mathcal{Z} [X] \mathcal{H} - [\mathcal{Z} X] \mathcal{H} = [X] \Lambda \]

so

\[ \mathcal{Z} \mathcal{X} + \mathcal{Y} = \]

\[ (\mathcal{X} \mathcal{X} + \mathcal{Y} \mathcal{X})_{\mathcal{Y} - \mathcal{X}} = \]

\[ (\mathcal{X} \mathcal{X} \mathcal{X}) \frac{\mathcal{Y} \mathcal{X}}{\mathcal{X} \mathcal{X} \mathcal{X}}_{\mathcal{Y} - \mathcal{X}} = \]

\[ \left( \frac{i \mathcal{Y}}{\mathcal{Y} \mathcal{X} \mathcal{X}} \right)_{\mathcal{Y} \mathcal{X} \mathcal{X}}^0 = \sum_{\mathcal{Y} \mathcal{X} \mathcal{X}} \frac{\mathcal{Y} \mathcal{X}}{\mathcal{X} \mathcal{X} \mathcal{X}}_{\mathcal{Y} - \mathcal{X}} = [\mathcal{Z} X] \mathcal{H} \]

Second moment is

\[ \mathcal{X} \mathcal{X} = \frac{i \mathcal{Y}}{\mathcal{Y} \mathcal{X} \mathcal{X}} \sum_{\mathcal{Y} \mathcal{X} \mathcal{X}}^0 = \]

Note that
\[
\begin{cases}
0 & \text{if } t < 0, \\
(\alpha e^{-\alpha t}) & \text{if } t \geq 0
\end{cases}
\] = (t)f

**Exponential Distribution**

with parameter \( \alpha > 0 \): probability density function

\( F(t) \)

Exponential distribution models the intrarrival time between two occurrences of events.

Exponential distribution models the interarrival time with a given time interval.

Poisson distribution models the number of events that occurred in a given time interval. (Substitution of nucleotide bases)

Applications of Poisson and of exponential distribution.
\[
\frac{n}{P(t)} \approx \text{[arrived within } t \text{ time]} \\
\text{time. Let } n \text{ be mean interarrival time, let } P(t) \text{ be small interval of time.}
\]

Exponential distribution models memoryless interarrival time.

\[
\frac{\lambda}{t} = [X] \Lambda
\]

and

\[
\frac{\nu}{t} = [X] E
\]

By integration by parts.

\[
\text{otherwise} \\
0 \quad 0 \\
\{0 \quad 1 - e^{-a(t-\epsilon)} \}
\]

cumulative density function.
\[
\begin{cases}
0 \forall \epsilon > 0 \\
0 \leq \tau \leq \mu - \epsilon - 1
\end{cases}
= (\tau) I
\]

Yields exponential distribution.

\[
\frac{\eta}{\tau^\epsilon - \tau} = 0
\]

Thus

\[
\left( \frac{\eta}{\tau^{0.1}} - 1 \right) \approx [\text{time within arrival}].I
\]

Take limit

\[
\left( \frac{\eta}{\tau^{0.1}} - 1 \right) \approx [\text{time within arrival}].I
\]

Assuming independence (memoryless),

\[
\frac{\eta}{\tau^{0.1}} - 1 \approx [\text{time } \tau^{0.1} \text{ within arrival}].I
\]
Algorithm

\[
\begin{align*}
\{ t > X \in [n/2] \} \cdot d &= \\
\left[ \frac{n}{t} - \frac{1}{2} < X \in [1] \right] \cdot d &= \\
\left[ \frac{n}{t} - \frac{1}{2} < X \right] \cdot d &= \\
\frac{n}{t} - \frac{1}{2} - 1 &= [t \text{ arrived in time }] \cdot d
\end{align*}
\]

so \( x - 1 = [x < X] \cdot d \)

Let \( X \) be uniformly distributed on \([0, 1]\).\[\frac{n}{t} = \frac{2}{t}\]

with mean interarrival time
Hypergeometric Distribution

\[
P(X = k) = \frac{\binom{m}{k} \binom{n-m}{u-k}}{\binom{n}{u}}
\]

that within time \( t \) there are exactly \( u \) arrivals is approximately

Fact: Given \( \frac{X}{1} \) as the mean interarrival time, the probability

```java
return (mean*Log(x));
```

\[
x = \text{double} \text{ rand}() \text{ RAND_MAX}
\]
Application: DNA segmentation algorithm

\[ d - 1 = b \cdot \frac{u}{x} = d \]

\[ \text{where} \]

\[ b, q, d \left( \frac{q}{w} \right) \approx (q, m, u) \eta \]

For large, hyperarithmetically binomial.

Binomial distribution: choosing \( m \) balls with replacement.

Replacement.

Hypergeometric distribution: choosing \( m \) balls without replacement.

\[ \frac{\binom{w}{u}}{\binom{q-w}{s-u} \binom{q}{x}} = (q, m, u) \eta = [q = X] \cdot \eta \]

Hypergeometric distribution